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Experimental Satellite Trajectory Analysis Using Decision-Based Robust Design

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The techniques for quality engineering can be used efficiently to evaluate the predicted performance of systems with a small number of simulations; this is especially important in the early stages of design of systems and parameter design when analysis methods are time consuming or expensive. Currently, the NASA Johnson Space Center employs Monte Carlo simulations to analyze trajectories of space vehicles. Accurate estimates of the system performance deviations due to parameter noise are obtained by a large number of simulations. This method is computationally expensive, especially if it is necessary to compare different designs. We introduce a method that is a variation of the robust design techniques proposed by Taguchi and can be applied to the trajectory analysis and the design improvement of a deorbiting satellite, the Life Sat vehicle. For the trajectory analysis, Monte Carlo simulations are replaced with an efficient simulation technique based on orthogonal arrays. We demonstrate that the two methods produce equivalent results; however, the decision-based robust design proposed here considerably reduces computational time for efficient design improvement. This would allow us to minimize variation in system performance from the designer-specified target values while concurrently maximizing the achievement of other design goals.

Nomenclature

A_{ref}	= reference area, m^2
c_d	= drag coefficient
$\boldsymbol{e}_R$	= position unit vector
$\boldsymbol{e}_d$	= drag unit vector
F	= F statistic
$\boldsymbol{F}_{\text{tot}}$	= vector representing total force applied to vehicle, N
g	= acceleration due to gravity, m/s^2
h, h_0, h_s	= height, standard height, reference height, m
k	= quality loss coefficient
M, Y_0	= target value
m	= vehicle mass, kg
M_e	= mass of Earth, kg
n	= number of observations (or simulations)
R_0, R	= radius from coordinate origin, Earth radius, m
SS_T, SS	= total sum of squares, sum of squares
v_r	= vector (magnitude) of relative velocity, m/s
Y_i	= ith observation of quality characteristic
Δ	= tolerance of uniform distribution
η	= signal-to-noise ratio
μ	= gravitation parameter or mean value, m^3/s^2
ρ, ρ_0	= air density, standard air density, kg/m^3
σ	= standard deviation of normal distribution

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Introduction

IN this paper we introduce an efficient approach for the trajectory analysis and the robust design of a satellite, the Life Sat. The proposed method is suitable for design problems that require a large number of time-consuming simulations to measure performance dispersions due to system noise. In rocket design, an important functional requirement is that the rocket must follow a predetermined trajectory. For the design of space vehicles, NASA currently determines trajectories using the SORT (Simulation and Optimization of Rocket Trajectories) program. The Life Sat is to deorbit and land within a desired target area in White Sands, New Mexico.[1] Deviations from the target (desired trajectory) will occur due to dispersions in vehicle and environmental parameters. These deviations make the achievement of design goals uncertain. In order to obtain accurate estimates of mean and variance of the longitude and geodetic latitude of the landing site for one proposed design, more than 1000 Monte Carlo simulations must be performed. This is computationally expensive, especially as it is often necessary to compare several designs or iterate to improve a single design. Therefore, the Monte Carlo method must be replaced with a more efficient method.

In order to verify the proposed method, the Life Sat simulation model is developed as a case study. Then Taguchi's concepts of quality, robust design, and orthogonal arrays (OAs) are introduced and discussed. The use of OAs forms the core of our approach for simulation reduction. The trajectory analysis results are discussed and a statistical comparison between the well-established Monte Carlo method and an OA-based simulation is performed to establish confidence in the results of the proposed method. The analysis of the trajectory simulation delivers the relative importance of the system parameters and enables the selection of important design variables. We discuss how to use the simulation method efficiently in making design decisions. The goal of the robust design process is to find factor levels that keep the trajectory on the desired target and minimize variations in the trajectory.

Life Sat Simulation Model

The trajectory analysis of the Life Sat is determined by the simulation of the uncontrolled deorbiting process. Depending on the initial state (i.e., speed and position) and vehicle and environmental parameters, the vehicle is driven by a combination of gravitational and aerodynamic forces. The initial state is selected in such a manner that the vehicle will land on target. But as a result of dispersions (noise) in vehicle and environmental parameters, the system performance is characterized by a dispersion, quantified by a value for mean and standard deviation. Currently NASA Johnson Space Center (JSC) uses a multivariate Monte Carlo simulation procedure for the trajectory analysis. In this method, each trial is composed of a randomly selected superposition of dispersion sources (which are uniformly or normally distributed). In order to obtain accurate estimates of mean and variance of the calculated response, the Monte Carlo method requires evaluation of the response under a large number of testing conditions. For the Life Sat example, each complete Monte Carlo simulation consists of 1109 trial simulations, a number that has been computed to provide 99.73% reliability.

The Life Sat space vehicle is modeled simply. In vector form, the equation for the total applied force is given by

$$F_{\text{tot}} = -m(\mu/R^2)e_R + \tfrac{1}{2}\rho v_r^2 A_{\text{ref}} c_d e_d \tag{1}$$

where $\mu = 3.98 \times 10^{14}$ m^3/s^2, the gravitation parameter of Earth, e_R a unit vector directed radially outward from Earth's center to the vehicle, v_r the velocity relative to the atmosphere, and e_d a unit vector in the direction of the drag (i.e., opposite the relative velocity vector). Both v_r and R are calculated from state vectors for speed and position and are expressed either in terms of Cartesian coordinates x, y, and z or in terms of longitude, geodetic latitude, and altitude for position and speed, flight-path angle, and azimuth angle for velocity.

The atmospheric density is necessary for calculating the aerodynamic force, and it largely determines the trajectory of the entering vehicle. As a first approximation, however, planetary atmospheres can be assumed to have the following exponential density–altitude distribution:

$$\rho \approx \rho_0 \exp[-(h - h_0)/h_s] \tag{2}$$

where $h_s = 8.563$ km, and $\rho_0 = 1.2$ kg/m^3, assuming that g is constant between h_0 and h. The distribution (dispersion) of many natural parameters can be assumed to be either uniform (U) or normal (N). The Life Sat vehicle model includes nine dispersed vehicle and environmental parameters. These are vehicle mass U, aerodynamic drag coefficient N, atmospheric density N, and six parameters describing the initial state N, three for the initial position and three for the initial velocity. A uniformly distributed parameter can be represented by the mean μ and the tolerance Δ and a normally distributed parameter by its mean μ and standard deviation σ. Tolerance or standard deviation is given as a percentage of the mean value. Details of the dispersions of the Life Sat parameters are given elsewhere.[2]

Dynamic systems are often analyzed through simulation, since it might be difficult, as for the satellite problem, to obtain an analytical solution. Starting at the initial state, we use numerical integration techniques to calculate the trajectory. Stochastic models are those in which relationships depend on distributed parameters. Outputs for a given set of inputs can be predicted only in a probabilistic context. Three methods to simulate such stochastic models are Monte Carlo simulation, Taylor series expansion, and OA-based simulation.

To validate our model implementation, we use initial state values that are chosen to be close to the ones documented by NASA JSC[1]: longitude, −106.65 deg; geodetic latitude, 44.3 deg; altitude, 121.92 km; initial velocity, 9946.5 m/s; initial flight-path angle, −5.88 deg; and initial azimuth, 180 deg. We expect the flight time to lie within the range of 280–330 s and the latitude of the landing position to be near 33.6 deg.[1] If all parameters are at their mean values, the simulation is called a nominal flight. The corresponding system performance is determined by the landing position (geodetic latitude, longitude) and two performance parameters: the peak acceleration during the flight, J_1, and the peak dynamic pressure, J_2. After the simulation of such a nominal flight is finished, data are obtained about the performance. Winds and the Coriolis force are neglected. Therefore the longitude remains at its initial value. The simulation results are longitude, −106.65 deg; geodetic latitude, 33.71 deg; altitude, 0.0 km; velocity, 112.8 m/s; J_1, 130.57 m/s^2; and J_2, 92,995.6 kg/ms^2. For performance parameter J_1 the corresponding g load is 13.3. This is compatible with documented values, which range from 11 to 15 g. There are no documented values for the performance parameter J_2. The values obtained for the nominal flight simulation make us confident that the model has been implemented properly; small deviations from documented values are due to simplifications.

Simulation Reduction Based on OAs

In this section, the concepts of quality and robust design are described.[3,4] OAs are introduced for efficient experimentation in general and for the simulation of noise factors in particular. A complete Monte Carlo simulation requires the randomly selected superposition of dispersion sources in many single simulations. In order to reduce this number, a description is found that is based on three selected levels of the distribution of each noise factor. Finally, by using OAs only a fraction of all possible factor combinations are simulated and the analysis of these simulations will deliver accurate estimates of the variations in system performance.

Introduction to Robust Design

Phadke,[3] following Taguchi,[4] measures the quality of a product in terms of the total loss to society due to functional variation and harmful side effects. Under the ideal conditions, losses would be zero; however, the greater the loss, the lower the quality. In design, quality is measured in terms of the fraction of the total number of possible simulations or experiments that do not achieve the desired results within a tolerance. However, this implies that all units that are within the tolerances of the requirements are equally good. In reality, a design whose predicted performance lies exactly on target is best. As the product's response deviates from the target, its quality becomes progressively worse. Therefore, it is necessary not to focus on meeting tolerances but on meeting the target. The quality loss $L(Y)$ for being off target is modeled by means of a quadratic quality loss function[3,4]:

$$L(Y) = k(Y - M)^2 \tag{3}$$

where Y is the quality characteristic, M is the target value for Y, and k is a constant, the quality loss coefficient. For maximum quality, the loss must be zero; the greater the loss, the lower the quality.

The factors that influence the quality of a product or process are represented in a P-diagram, where the P represents the product or process. A number of parameters (also known as factors) influence the quality characteristic Y of a system. Three types are distinguished. Signal factors M are set by the user or operator of the product to express the intended value for the product's response (target value). Noise factors x are not under the designer's control; they lead to quality loss. Parameters whose levels are difficult or expensive to control may also be considered to be noise factors. Levels of noise factors change from product to product, from one environment to another, and from time to time. Control factors z are parameters that can be specified by a designer so that system performance reaches the targeted values.

Taguchi proposes an engineering approach to quality. This is somewhat different than the approach to similar problems proposed by the statistical community in the United States. Using Taguchi's approach, a designer is required neither to know nor to care what causes variation in noise parameters. This is a short-term solution. Box[5] proposes rather that statistical experimentation be performed in order to identify and eliminate the sources of variation. We believe that there are appropriate places for each approach; indeed, here we use a combination of the philosophies.

How Can Quality Loss be Avoided?

Taguchi has suggested using a signal-to-noise ratio (S/N) as a predictor of quality loss after making certain simple adjustments to the system's function.[3,4] Thus a set of observations is converted into a single number and is used as the objective function to be maximized in robust design.[2,6–8] Phadke and Taguchi recommend maximization of the S/N:

$$\eta = 10 \log_{10}\left(\frac{\bar{y}^2}{\text{MSD}}\right) \tag{4}$$

where $\bar{y}$ is the mean value of the signal intensity, i.e., of the response and MSD is the mean-squared deviation:

$$\text{MSD} = \left[(Y_1 - Y_0)^2 + (Y_2 - Y_0)^2 + \cdots + (Y_N - Y_0)^2)/N\right]$$

where the Y_i are the actual system responses, Y_0 the target value, and N the number of trials; η is expressed in decibels. Equation (4) may also be written as

$$\eta = 10 \log_{10}(\bar{y}^2) - 10 \log_{10}(\text{MSD}) \qquad (5)$$

If the mean of the observations is adjusted to the target value, maximizing η is equivalent to maximizing Eq. (4) because the value is dependent on the deviation (MSD), too. However, there are various mathematical difficulties/requirements associated with the use of η.[5] We have chosen to address individually the issues of maximizing the intensity of the signal on target (adjusting the mean to the target value) and minimizing the variation around this target value; these become separate goals under a designer's control in decision-based-design.[6] As with the other goals, a designer assigns priorities to these goals based on his or her perception of the requirements of a specific problem; hence this method can be called decision-based robust design.

As the Life Sat has a specified target area, we focus on the situation that Taguchi calls "the nominal is the best." This situation is the most mathematically rigorous.[5] We use two measurements of quality, based on a target value, Y_0, the degree to which the mean of the sampling population meets Y_0 and the quality characteristic Θ, which is defined[8] as

$$\Theta = -10 \log_{10}(\text{MSD}) \qquad (6)$$

The design requirements for the Life Sat can easily be stated in terms of the factors used in the P diagram. The quality characteristic for the Life Sat is its ability to follow a desired trajectory and land at a desired target. The response of a proposed design is its actual trajectory including deviations from the target. Noise factors are environmental and vehicle parameters. Control factors are related to the vehicle itself and are specified by the designer, e.g., mass, velocity, dimensions, and/or coefficients. In this case signal factors are represented by the initial state of the vehicle. In the Life Sat example we are concerned with the distribution of the landing position within specified tolerances on the target. Following the robust design approach, we want to decrease the variation around the mean of the landing position and also minimize the bias between the mean and the target.

Factorial Experiments with OAs

In order to perform experiments in any field of research usually factor levels are defined and various combinations of these levels are analyzed. A level is a selection of a factor value within a possible range. For engineering problems involving multiple factors, the number of possible combinations becomes extremely large. To reduce the number of experiments to a practical level, a smaller set must be chosen. The design of experiments methods (DOEs) offers a structured way to select a set of experiments. Taguchi[4] uses a special class of fractional factorial experiments, OAs, to describe a large number of experimental situations. The power of an OA is the ability to evaluate several factors with a minimum of experimentation. A great deal of information is obtained from a few trials, but there are also limitations and weaknesses. One has to be aware that Taguchi's method usually identifies only main effects and ignores interactions.[9,10] Standard OAs are available, which are defined as L_i arrays, where i is the number of experiments required. Each factor is assigned to a column of the OA in which the factor levels are listed. The choice of the OA depends on the number of factors and their levels. In Table 1 the numbers of experiments required for a full factorial design and an OA design are compared. The specified number of factors is also the number of columns in the corresponding OA. We have chosen three levels for each parameter in the Life Sat model. Having nine factors on three levels for the noise simulation, the L_{27} orthogonal array is required, which can be used for a maximum of 13 factors. But for the robust design process, having three design variables on three levels, an L_9 OA can be used. Further information about the use of OAs is available.[11]

Table 1 Full factorial design comparison with OA design

Factors	Factor levels	Factorial design, number of experimental trials required	OA design, number of experiments/trials: Required	OA used
3	2	8 (2^3)	4	L_4
7	2	128 (2^7)	8	L_8
15	2	32,768 (2^{15})	16	L_{16}
4	3	81 (3^4)	9	L_9
13	3	1,594,323 (3^{13})	27	L_{27}

In an OA, for any pair of columns, all combinations of factor levels occur an equal number of times. This is the balancing property.[3] For space missions, accuracy and confidence are essential. Therefore, three levels for each noise factor have been selected. For three levels of the noise variables, the following level values are recommended[3]: for a normally distributed three-level variable with a mean μ_i and a variance σ_i^2, the levels $\mu_i - \sqrt{1.5}\sigma_i$, μ_i, and $\mu_i + \sqrt{1.5}\sigma_i$ should be used; for uniformly distributed variables with mean μ and tolerance Δ, the three-factor levels are $\mu - \Delta$, μ, and $\mu + \Delta$, respectively.[3,12] This selection of noise levels to represent their distribution is rather critical, and changes will result in very different estimates of the response distribution.[2]

Predicted Performance of Proposed Designs

The Monte Carlo method, with 1000 simulations, provides a baseline for comparison of both simulation methods. We have performed two Monte Carlo runs with different seed numbers and thus have two sets of data. Two sets of data for OA experiments are also obtained by assigning the noise factors to different columns in the L_{27} OA. The initial state is chosen so that the selected parameter values result in a landing position that is close to the desired target area of −106.65 deg longitude and 33.6 deg geodetic latitude (Fig. 1). Each point in Fig. 1 represents the vehicle's landing position for a single experiment, that is, for the output of one simulation of the flight trajectory. The Monte Carlo simulation has the greatest number of points concentrated in an area between −106.52 and −106.78 deg longitude and 31.7 and 35.5 deg geodetic latitude. Assuming that 1 deg longitude ≙ 111.2 km and 1 deg geodetic latitude ≙ 111.2 km, most of the points lie approximately within a 30 × 420 km rectangle. A footprint from an OA-based simulation is presented in Fig. 1. To assess the differences between the two methods, it is necessary to compare flight statistics for the landing range for each simulation method.

Performance statistics are presented in Table 2. Since this is a nonlinear model, it is impossible to estimate means in advance. Hence, values for the nominal flight are given. In the remaining rows the results for two Monte Carlo simulations (each with 1000 trials) and two OA simulations (with 27 trials) are presented. Comparing mean values for the longitude, the two methods are apparently equivalent. For all four simulations, the values for the standard deviation range from 0.0541 to 0.0582 deg; this represents approximately 6 km. For the geodetic latitude, there is a slight difference in the mean value between the nominal flight and the other simulations; the standard deviation represents approximately 90 km. To verify the model and

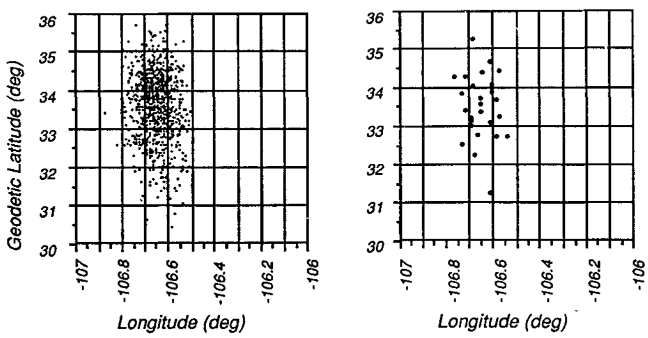


Fig. 1 Footprints of Monte Carlo simulation and OA-based simulation.

Table 2 Landing range and performance parameter statistics

Method	Longitude		Geodetic latitude		Maximum acceleration		Maximum dynamic pressure	
	Mean μ, deg	Std. dev. σ	Mean μ, deg	Std. dev. σ	Mean μ, m/s^2	Std. dev. σ	Mean μ, kg/ms^2	Std. dev. σ
Nominal flight	−106.650	—	33.625	—	141.569	—	100,992.8	—
Monte Carlo 1	−106.652	0.0541	33.591	0.808	141.490	10.164	101,092.0	7848.7
Monte Carlo 2	−106.651	0.0582	33.581	0.825	141.798	10.733	101,289.0	8329.1
OA 1	−106.650	0.0548	33.582	0.813	141.455	10.355	100,510.0	8452.9
OA 2	−106.650	0.0547	33.582	0.813	141.457	10.354	100,511.0	8434.5

Table 3 Distribution of means and standard deviations for 10 Monte Carlo simulations and 10 orthogonal array simulations

	Monte Carlo (1000)		Orthogonal array (27)	
	Latitude μ, deg	Latitude σ, deg	Latitude μ, deg	Latitude σ, deg
	33.588	0.642	33.592	0.641
	33.603	0.678	33.593	0.633
	33.599	0.698	33.593	0.619
	33.591	0.654	33.590	0.671
	33.596	0.648	33.592	0.644
	33.588	0.663	33.591	0.642
	33.601	0.672	33.591	0.654
	33.606	0.657	33.590	0.661
	33.601	0.669	33.591	0.649
	33.574	0.660	33.592	0.637
Average	33.595	0.664	33.592	0.645
Standard error	0.0096	0.0161	0.0011	0.0147

compare Monte Carlo and OA simulations, we are interested in the difference of the outputs. For maximum acceleration the statistics are approximately equivalent. For maximum dynamic pressure, the mean values for OAs are smaller and standard deviations are larger, but they are of the same magnitude. In general, differences between the two Monte Carlo simulations are much greater than between two OA-based simulations.

Since similar results for both simulations of mean and standard deviation have been obtained for several output parameters, the question arising now is: Do the statistical results for OA simulations represent the same population as the results obtained by Monte Carlo simulation? To answer this question, geodetic latitudes for 10 different Monte Carlo simulations have been compared with 10 OA simulations (Table 3). A t test is used[13] to determine whether or not the means and standard deviations come from the same populations; the results suggest that the populations are the same at least 97.5% of the time. This is important because it implies that the results from OA-based simulations are identical to those obtained by the Monte Carlo method.

Parameter Effects on System Performance

Monte Carlo Method

In order to determine which noise parameters influence system performance most significantly, the nine parameters of the initial state are dispersed (Table 4) before they are transformed into the Cartesian coordinate system. Each is varied singly for a 1000-point Monte Carlo simulation (because 1000 simulations have been performed before) and the total sum of squares for that factor is calculated as

$$SS_T = \sum_{i=1}^{N} \left(y_i - \frac{T}{N} \right)^2 \tag{7}$$

where T is the sum of all observations and N the number of experiments. The contribution of each factor to the output variation is calculated by dividing the so-called pure sum of squares by the total sum of squares (Table 5). The pure sum of squares depends on the degrees of freedom of a factor and the error variance. Only seven parameters are shown in Table 5. The variation of the density has the greatest influence on the variation of the geodetic latitude; velocity is the next largest contributor. Vehicle mass, altitude, and flight-path angle have approximately equal influences. The contribution of the drag coefficient is insignificant. The sum of squares for the azimuth and longitudinal initial position are not significant and are summarized in "other" and become part of the error term when the OA results are analyzed.

Table 4 Parameter statistics

Parameter	Mean μ	Standard deviation σ
Initial velocity	9,946.5 m/s	0.667%
Longitude	−106.65 deg	0.1 deg
Geodetic latitude	44.2 deg	0.05 deg
Vehicle mass	1560.4 kg	1.667%
Density	1.2 kg/m^3	10%
Drag coefficient	0.6679	1.667%
Altitude	121,920.0 m	250.0 m
Flight-path angle	−5.88 deg	0.25%
Azimuth	180.0 deg	0.333%

Table 5 Factor contribution to variation in geodetic latitude for Monte Carlo simulations

Parameter	Total sum of squares	Percent contribution
Density	27.901 kg/m^3	45.10
Vehicle mass	2.773 kg	4.49
Vehicle drag coefficient	0.814	1.32
Initial velocity	13.975 m/s	22.60
Flight-path angle	2.765 deg	4.47
Altitude	2.902 m	4.69
Latitude	3.366 deg	5.44
Other	0.813	1.31

OAs

By using OAs, all parameters are varied simultaneously. Similarly to the results obtained using Monte Carlo simulation, the atmospheric density has the greatest contribution of approximately 50%, followed by velocity with 25%. The third largest contributor is proposed vehicle mass with 9%. The drag coefficient has the smallest effect with 1.3%. Having identified factors contributing to the variation in system performance, a designer's next step is to select control factors. This selection should be based both on their importance and on convenience. Then the selected factors are adjusted to increase design quality.

Robust Design of Life Sat Vehicle

The quality characteristic for the Life Sat vehicle is defined as its ability to follow a desired trajectory and land at a specified target position. The greater the deviation from the target, the higher the quality loss. Although the vehicle's nominal design values are chosen so that the Life Sat is predicted to land on target, noise factors cause variations in its anticipated landing position.

Assume a designer has chosen three control factors: vehicle mass, initial velocity, and flight-path angle. Assume further that he or she will perform a robust design to find the best combination of parameter values to make the vehicle land as closely as possible to the target for average performance and also to obtain the least variation around the target. From previous information, the mass can vary from 1460 to 1660 kg, velocity from 9846.5 to 10046.5 m/s, and flight-path angle between −5.78 and −5.98 deg. Having three factors with three levels each, the L_9 OA is used. This standard

Table 6 Standard L_9 OA

Experiment	A	B	C	D
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

Table 7 Factor levels for robust design of Life Sat vehicle

Factor	Level 1	Level 2	Level 3
Vehicle mass, kg	1,460.0	1,560.0	1,660.0
Velocity, m/s	9,846.5	9,946.5	10,046.5
Flight-path angle, deg	−5.78	−5.88	−5.98

Table 8 Experiments for robust design of Life Sat vehicle

	Geodetic latitude, deg				
Experiment	μ	σ	$\eta(T_1)$	$\eta(T_2)$	$\eta(T_3)$
1	33.572	0.219	11.78	**12.58**	9.07
2	33.692	0.215	**13.16**	10.70	6.94
3	33.817	0.211	12.23	8.34	5.04
4	33.755	0.209	13.14	9.57	5.98
5	33.870	0.205	11.37	7.42	4.32
6	33.092	0.246	3.63	6.34	9.73
7	33.933	**0.200**	10.19	6.40	3.55
8	33.178	0.237	4.80	7.90	11.33
9	33.311	0.232	6.83	10.37	**12.49**

OA is shown in Table 6. There are four factors (A, B, C, D) on three levels (1, 2, 3). A total of nine experiments is required. The selected factor levels are lower bound, middle value, and upper bound, corresponding to levels 1, 2, and 3 in Table 7.

Assigning the three factors to the first three columns of L_9, nine experiments are performed. Each of these nine experiments is performed using the noise factor dispersions. These nine experiments of L_9 are called an inner array and represent different combinations of factor levels under a designer's control. For each experiment in the inner array it is necessary to estimate the system's sensitivity to noise factors. This is done by applying an outer array of 27 experiments in which the uncontrolled dispersions of the noise factors are simulated. Note that this also includes the dispersion of the selected control factors around each of the levels in L_9. Hence the inner array is used to represent different designs while the outer array is used to simulate the variation in system performance for each of these designs.

Previously the target value for the geodetic latitude was 33.6 deg. In this section, the S/N η is calculated for three new target values for the geodetic latitude, $T_1 = 33.7$ deg, $T_2 = 33.5$ deg, and $T_3 = 33.3$ deg, in the order to show the influence of the selected target. First, we calculate the mean and the standard deviation of the geodetic latitude, which are independent from the target. The results of the simulation of the experiments are shown in Table 8.

The boldface numbers in Table 8 represent the best designs with respect to the standard deviation and η. Notice that the experiment with the smallest standard deviation does not correspond to the highest η because the mean value is far from the target. In what follows, T_1 is used as the target in order to illustrate the method.

To study the average output sensitivity of a factor on one level, e.g., on level 1, the mean of the response for all experiments is calculated using the factor at level 1. For these experiments the average geodetic latitude is $(33.57 + 33.76 + 33.93 \text{ deg})/3 = 33.75$ deg. Similarly mean values are calculated for levels 2 and 3 and for the other factors. The results are shown in Table 9 for the mean of the geodetic latitude, the standard deviation, and two of the η's. The three control factors are denoted as A, B, and C, where A is vehicle mass, B is velocity, and C is flight-path angle, and the factor levels are referred to as A_1, for factor A at level 1, etc. The values in boldface type represent the best designs.

Table 9 Level means for system performance

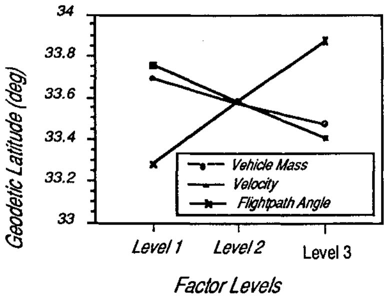
Factor	Level 1	Level 2	Level 3
a) Level means for latitude, deg			
A: Vehicle mass, kg	33.694	33.572	33.474
B: Velocity, m/s	33.753	33.58	33.407
C: Flight-path angle, deg	33.281	33.586	33.873
b) Latitudinal deviation from target, bias			
A: Vehicle mass, kg	**0.006**	0.128	0.226
B: Velocity, m/s	**0.053**	0.120	0.293
C: Flight-path angle, deg	0.419	**0.114**	0.173
c) Level means for standard deviation			
A: Vehicle mass, kg	**0.215**	0.220	0.223
B: Velocity, m/s	**0.209**	0.219	0.230
C: Flight-path angle, deg	0.234	0.219	**0.205**
d) Level means for η			
A: Vehicle mass, kg	**12.39**	9.38	7.27
B: Velocity, m/s	**11.70**	9.78	7.57
C: Flight-path angle, deg	6.74	11.05	**11.26**

The mean values for the geodetic latitude are shown in part a of Table 9. The smallest deviation from the target value corresponds to factor levels A_1, B_1, and C_2, respectively (Table 9, part b). Similarly, in order to achieve the lowest standard deviation in the system performance (the response), factor levels A_1, B_1, and C_3 should be chosen (Table 9, part c). In Table 9 (part d) both these quality aspects are combined to yield the S/N η. From this table it can be seen that the factors corresponding to the best system performance with respect to both the bias and the sensitivity to noise are A_1, B_1, and C_3. The average sensitivity of the system's performance (the geodetic latitude) is shown as a function of the various factor levels (Fig. 2a), and the system's sensitivity to variations is shown in Fig. 2b. The flight-path angle is the control factor with the greatest influence on the vehicle's performance with respect to latitudinal dispersion. The relative influence of each control factor can also be estimated by computing the sums of squares, but this yields no information about the signs of the slopes.

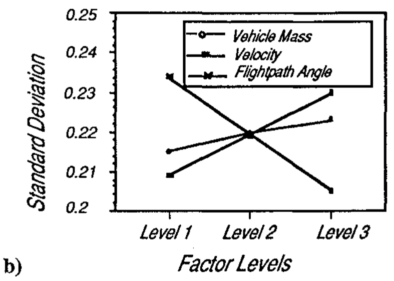
How can we now find an improved design from this information? We compare three different strategies: 1) use the experiment with the smallest variation and adjust the mean to the desired target value, 2) find the factor levels associated with the least variation and perform an experiment to verify that these are appropriate, and 3) find the factor levels associated with the least variation and perform an experiment to verify that these are appropriate and then adjust the mean on target. In many systems, signal factors can be used for this purpose. To adjust the mean on target the initial value for the geodetic latitude is used, and it is adjusted by the value of the bias between the mean and the target (Fig. 2). By changing the initial geodetic latitude value, only a proportional bias in the final geodetic latitude is expected. This will be confirmed in the design process.

Our initial strategy is to take the experiment with the smallest variation and then adjust the mean on target. The experiment with the smallest standard deviation is experiment 7 with factor levels A_3, B_1, and C_3. As can be seen from Table 8, the deviation from the target mean of $T_1 = 33.7$ is $33.933 \text{ deg} - 33.7 \text{ deg} = 0.233$ deg. The initial geodetic latitude behaves like a scaling factor as a change in this value results in an almost proportionate change in the final value for the geodetic latitude. In this case, the initial geodetic latitude is changed from 44.2 deg to $44.2 \text{ deg} - 0.233 \text{ deg} = 43.967$ deg. To verify this, a simulation is performed with factor levels A_3, B_1, and C_3. The following results for the mean, the standard deviation, and η are obtained: $\mu = 33.6998$ deg, $\sigma = 0.200$, $\eta = 13.819$. The mean is on target (33.7 deg) and the standard deviation is unchanged, whereas η is greater than all previous values. Therefore, this is an improved design with higher quality.

In the second strategy, factor levels are selected that have with the highest average η. From Table 8, level 1 for factor $A(A_1)$ has



a)



b)

Fig. 2 a) Average geodetic latitude for different factor levels and b) average sensitivity of standard deviation to factor levels.

$\eta = 12.39$ and a standard deviation of $\sigma = 0.215$. These average values are preferred with respect to the quality characteristics to lower values of the S/N and higher values of the standard deviation for levels A_2 and A_3. Similarly level 1 for factor $B(B_1)$ and level 3 for factor $C(C_3)$ are selected. In the verification experiment using levels A_1, B_1, and C_3, the following results are obtained from the simulation with OAs: $\mu = 34.12$ deg, $\sigma = 0.195$, $\eta = 6.705$.

The mean for the geodetic latitude is $\mu = 34.12$ deg and is thus 34.12 deg − 33.7 deg = 0.42 deg away from the desired target. Thus η is low, although the standard deviation is small. The combination of factor levels with the lowest average standard deviation has resulted in an even lower value of $\sigma = 0.195$. Therefore, it is important to verify the suggested factor levels. Taking the factor C at level C_3 with an average mean value for the geodetic latitude of $\mu = 33.87$ deg brings us to the new mean of $\mu = 34.12$ deg. Thus, level C_2 results in a better value. Adjusting the mean on target, as suggested for the third strategy, the initial latitude is changed according to the bias of 0.42 deg to 44.2 deg − 0.42 deg = 43.78 deg, resulting in the following values from the simulation: $\mu = 33.70$ deg, $\sigma = 0.1946$, $\eta = 14.037$. This best design, obtained by using the suggested factor levels A_1, B_1, and C_3, has the following values for the design parameters: vehicle mass 1460 kg, velocity 9846.5 m/s, flight-path angle −5.98 deg. It was necessary to use the initial geodetic latitude as an adjustment factor to get the mean on target. Without the ability to adjust the mean on target, we would have chosen factor levels that give the highest η for one experiment, experiment 2 in Table 7, in which $\eta = 13.16$.

All of these results and information are obtained with only 9 experiments. Of course, it was necessary to perform 27 simulations in the outer array to simulate the effects of the noise factors. This is done for each of the 9 experiments. This makes it a total of $27 \times 9 = 243$ trajectory simulations, which is about one-quarter (25%) of the number of Monte Carlo simulations (more than 1000) necessary to evaluate one single design. The computational time by using OAs to simulate the noise factors is 95% (27 compared to 1000 experiments) less than time required for Monte Carlo simulations.

Conclusions

In this paper we have shown how the selection of three noise factor levels to represent their distribution can be efficiently used for a trajectory analysis. Accurate estimates of the system performance are obtained. As the number of necessary simulations is greatly reduced, it is possible to explore more possible designs. Although a design is found that has a minimum variation around the target, this design is based on factor levels. Further refinement of the design might be obtained by narrowing the ranges and repeating the simulation rather than exploring the entire design space with large "one-shot" experiments[9] to acquire all needed information. One of the keys to dealing with this problem is to use sequential strategies.

In Sprow's[10] view, Taguchi has increased our awareness of the possibilities, but his methodology has some problems, e.g., Taguchi downplays interactions. It is important to know that the Taguchi method cannot be used for everything. Other designs of experimental methods (e.g., the response surface methodology) can be used for analyzing problems in which several independent variables influence a dependent variable or response. The goal is to optimize this response.[11] As a result, not only the main effects that influence a response are detected, but also interaction effects.

Our way of improving the robust design technique is to incorporate it into a multiobjective decision support system: decision-based design. Building on this, we have implemented the robust design technique into the compromise decision support problem based on the satellite case study presented in this paper.[6] The trajectory is analyzed using this approach to simulate noise factors, but the robust design optimization is based on a goal programming technique.

We are also investigating the use of OAs and robust design techniques in a concept exploration model (CEM), which would be used to generate quickly a large number of possible designs. In its simplest form, a CEM consists of a mathematical model of the system to be explored, a set of control factors to be varied, a point generator, and a set of goals and achievement functions by which the system performance can be measured. In previous applications of CEMs the set of control factors has usually been those variables believed to have the greatest influence on the system, the system model has been either a calculated model or an optimization model of the system, and the point generator has been an algorithm producing either random or grid points covering the design space.[14–17]

Usually it is difficult to determine a priori which factors influence the design the most. This is especially true for original design. Therefore, it is advantageous to retain as many factors as possible as candidates for variation. However, to maintain the same density of points in the design space, the number of points that must be calculated will increase rapidly with the number of factors and hence decrease the efficiency. To counteract this explosion in the number of points required, we propose the use of OAs as an alternative to random or grid point generators. The use of OAs allows us to place systematically relatively few points in the design space. By exploiting the inherent properties of OAs, the system's performance at all points in the design space corresponding to a factorial design should be predictable, thus avoiding the necessity of carrying out the actual calculation of system performance at these points.

Further, this would allow us in a very early stage of design to seek design solutions where the performance is relatively insensitive to noise factors, that is, those factors outside the control of the designer.

We believe that in the future we need to focus more closely on the very early stages of the design process, since decisions made here have a large impact on the final design. Supporting these decisions by the combined use of robust design methodology and decision support problems seems to us a promising way forward.

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